Pen And Paper Level: Intermediate

We will study some RG flows in 3 dimensions. Consider two decoupled Ising models. Denote the energy operator of the first model by $\epsilon_1$ and the energy operator of the second model by $\epsilon_2$.

1.) Let us deform the theory by

$$\mathcal{L} = \mathcal{L}(Ising1) + \mathcal{L}(Ising2) + \lambda\epsilon_1\epsilon_2 \ . \tag{1}$$

Is this deformation relevant? what is its dimension?

Our goal is to argue that this flows to the $O(2)$ model and to compute some of the properties of the $O(2)$ model using *leading order* conformal perturbation theory. You can use any result in the literature about the Ising model (anomalous dimensions, OPE coefficients, etc).

2.) Compute the one loop correction to the beta function of $\lambda$ in conformal perturbation theory. Is there an infrraed fixed point? What is the dimension of the operator multiplying $\lambda$ there?

3.) The symmetries of the above model (1) are given by some subgroup of $O(2)$. Identify the subgroup.

4.) Identify the operator multiplying $\lambda$ in the $O(2)$ model. Check if you get a reasonable agreement.

4.) What would you do to compute the dimension of $\epsilon_1$ in the new fixed point? This should match with the dimension of the energy operator in the $O(2)$ model. Think before answering.